Journal of Alloys and Compounds, 213/214 (1994) 460–464
JALCOM 4043

Production of heavy actinides in incomplete fusion reactions

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Abstract

We present preliminary results of calculations by the phenomenological model of the estimated yield of some heavy actinide isotopes. It is assumed that these isotopes are produced as a result of multinucleon transfers followed by neutrons and charged particle emission A.S. Iljinov and E.A. Cherepanov (*Proc. Int. Symp. on Synthesis and Properties of New Elements, Dubna, 1980,* Joint Institute for Nuclear Research, Dubna, 1980, D7-80-556, p. 29). The yield $P_{Z,N}(E^*)$ of primary excited actinides is found using the model of N.V. Antonenko and R.V. Jolos (*Z. Phys. A, 338* (1991) 423). Absolute cross-sections for different binary reaction channels are obtained by summing the cross-sections for all subchannels with an appreciable yield according to J. Wilczynski *et al.* (*Phys. Rev. Lett., 45* (1980) 606).

1. Introduction

Investigations have provided an appreciable amount of new information on heavy ion induced reactions. In the incomplete-fusion reactions nucleons are exchanged between two nuclei. Because of this the excitation energy of the target-like product is low and the amount of evaporated particles is not large. Therefore, incomplete-fusion reactions were studied extensively in order to explore the possible production of isotopes of heavy actinides (see, for example, ref. 1).

To estimate the production cross-sections of different nuclides we have to calculate preliminary isotope distribution and to obtain the final distribution taking into account particle evaporation. After this two-step calculation we can compare theoretical and experimental results. Since our microscopic calculation allows us to obtain the preliminary isotope distribution then calculated isotope production cross-sections are in agreement with the Q_{gg} systematics. The slope of the Q_{gg} systematics is the initial information for the calculation of the final isotope distribution within the statistical evaporation model.

The evaporation process is calculated within the statistical code [2]. Bearing in mind that in heavy ion reactions the nuclei formed have large angular momentum $I \gg 1\hbar$, we choose the quasi-classical version of this model, in which the spins S of the emitted particles are neglected and the angular momenta I_i and I_f of the original and final nuclei, as well as the particle orbital momentum l are considered to be classical vectors. We take into account the emission of γ rays, neutrons, protons, ^{2}H, ^{3}H, ^{3}He, α particles, and fission.

2. Method of calculation

2.1. Microscopic treatment of transfer reactions

It was found that, as the neutron number of the projectile increases [3], the corresponding maximum of the mass distribution of the target-like elements increases as well at $Z_T > Z_{T_0}$ (Z_{T_0} is the initial atomic number for the target). However, this shift of the nuclide distribution does not coincide with the change in neutron number in the projectile. For instance, the eight neutron excess of ^{48}Ca relative to ^{40}Ca did not increase the maximum of the isotopic distributions by 8 mass units in the collisions with ^{248}Cm. Shifts to heavier masses are 2–3 units only for the elements Bk, Cf, Es and Fm. The reason for this effect could be connected with the mechanism of formation of the preliminary isotope distribution.

Experimental data had been obtained which demonstrated the influence of the shell effects on the multiple nucleon transfer process. The interpretation of this effect required the formulation of the microscopic model of the process. A variant of that approach has been suggested in ref. 4. It was shown that the treatment of the mass distributions on microscopic grounds requires the consideration of the proton and neutron transfer simultaneously.

The main ingredient of the model suggested in ref. 4 is the single-particle level scheme. This is in contrast

SSDI 0925-8388(93)04043-4

with the transport theories which are based on the use of the driving potential [5]. The isospin dependence of the single-particle energies which is due to the total symmetry energy is taken into account. It is important for the correct description of the proton and neutron transfer.

We consider a dinuclear system after the relative motion kinetic energy damping. Let $P_{Z,N}(t)$ be the probability to find the system at the moment t in the state with mass $A=N+Z$ and charge Z of the light fragment. In ref. 4 the master equation for $P_{Z,N}(t)$ has been obtained:

$$\begin{aligned} P_{Z,N}(t) = {} & \Delta^{(-,0)}_{Z+1,N} P_{Z+1,N}(t) + \Delta^{(+,0)}_{Z-1,N} P_{Z-1,N} \\ & + \Delta^{(0,-)}_{Z,N+1} P_{Z,N+1}(t) + \Delta^{(0,+)}_{Z,N-1} P_{Z,N-1}(t) \\ & - (\Delta^{(-,0)}_{Z,N} + \Delta^{(+,0)}_{Z,N} + \Delta^{(0,-)}_{Z,N} + \Delta^{(0,+)}_{Z,N}) P_{Z,N}(t) \end{aligned} \tag{1}$$

Here the transport coefficients have been calculated microscopically:

$$\Delta^{(\pm,0)}_{Z,N} = \frac{1}{\Delta t} \sum_{P,T} |g^{Z}_{PT}|^2 n^{Z}_{T,P}(\tau)[1-n^{Z}_{P,T}(\tau)] \times \frac{\sin^2[(\Delta t/2\hbar)(\tilde{E}_{P_Z}-\tilde{E}_{T_Z})]}{(\tilde{E}_{P_Z}-\tilde{E}_{T_Z})^2/4} \tag{2}$$

$$\Delta^{(0,\pm)}_{Z,N} = \frac{1}{\Delta t} \sum_{P,T} |g^{N}_{PT}|^2 n^{N}_{T,P}(\tau)[1-n^{N}_{P,T}(\tau)] \times \frac{\sin^2[(\Delta t/2\hbar)(\tilde{E}_{P_N}-\tilde{E}_{T_N})]}{(\tilde{E}_{P_N}-\tilde{E}_{T_N})^2/4} \tag{3}$$

In this model the independent single-particle proton (index Z) and neutron (index N) transfers are considered. The matrix elements g_{PT} and single-particle energies are determined in accordance with the following expressions:

$$\begin{aligned} \tilde{E}_{P_Z} &= E_{P_Z} + \frac{Z_T e^2}{2R} \\ \tilde{E}_{T_Z} &= E_{T_Z} + \frac{Z_P e^2}{2R} \\ \tilde{E}_{P_N} &= E_{P_N} \\ \tilde{E}_{T_N} &= E_{T_N} \\ g^{Z}_{PT} &\approx 0.5\langle P_Z | U^{Z}_{P} + U^{Z}_{T} | T_Z \rangle \\ g^{N}_{PT} &\approx 0.5\langle P_N | U^{N}_{P} + U^{N}_{T} | T_N \rangle \end{aligned} \tag{4}$$

Here $|P_{Z,N}\rangle$ and $|T_{Z,N}\rangle$ are the single-particle state vectors of the non-interacting nuclei for projectile and target respectively, R is the distance between fragment centres, and Z_P, Z_T are the proton numbers of the fragments. The mean single-particle potentials $U_{P,T}$ include both the nuclear and Coulomb (for protons) fields. Excluded in eqns. (4) are the non-diagonal matrix elements that generate the nucleon transitions between the single-particle levels in the same nucleus. This leads to the Fermi surface diffusion. We assume that this effect is taken into account by the phenomenological temperature occupation numbers $n^{Z,N}_{P}(\tau)$ and $n^{Z,N}_{T}(\tau)$. The temperature τ is determined by the system excitation energy. We fixed Δt in eqns. (2) and (3) to be equal to 2×10^{-22} s. This corresponds to the relaxation time of the nuclear average field.

Solving eqn. (1) we obtain the preliminary nuclide distribution. It should be noted that these distributions are presented in relative units. There are some theoretical arguments [6] supporting the Q_{gg} systematics that are connected with the assumption of partial statistical equilibrium. However, in the microscopic treatment we did not assume that statistical equilibrium is established in the system. However, it was shown in ref. 4 that calculated isotope production cross-sections are in a good agreement with Q_{gg} systematics. Because of this the following parametrization of isotope production probabilities can be used [6]:

$$Y(i) \propto \exp[(Q_{gg} - \delta + \Delta E_c)/T_0] \tag{5}$$

where ΔE_c is the change of the Coulomb interaction energy due to the charge transfer, δ is the so-called "non-pairing" energy correction, the parameter T_0 characterizes the slope of the exponent in eqn. (5), $Q_{gg}=M_i+M_t-(M_H+M_L)$, $\Delta E_c=(Z_iZ_t-Z_LZ_H)e^2/R_c$, and Z and M are atomic numbers and masses of the nuclei of the dinuclear system before and after transfer. The calculation results of ref. 4 allow us to say that there is no equality between T_0 and thermodynamic temperature τ. Therefore, the microscopic model gives an exponent slope which is used in subsequent statistical evaporation calculations. The parameter T_0 depends on τ and interaction time. Because of the smallness of the proton multiple evaporation the interaction time can be estimated according to the experimental charge variance. The results of calculations of some reactions by this model are presented in Fig. 1.

2.2. Sum-rule model

The term incomplete-fusion reaction was suggested for reactions in which the major part of the projectile transferred to the target. We used the "sum-rule" model which was suggested in ref. 7 for calculating cross-sections of yield fragments in transfer reactions. According to this model incomplete-fusion reactions are localized in successive l windows above the critical angular momentum for a complete fusion, in a sequence beginning with the capture of the heaviest fragment of a projectile, followed by the capture of lighter fragments at higher angular momenta [7]. For the cut-

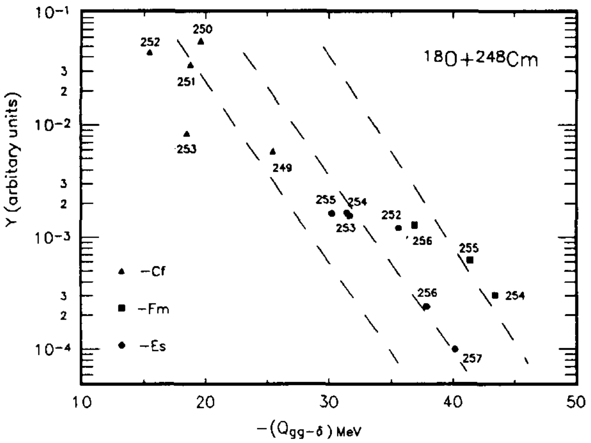

Fig. 1. Results of calculations using the model in section 2.1.

off in angular momentum space a smooth cut-off is taken:

$$P_l=\{1+\exp[(l-l^i_{\rm crit})/\Delta_l]\}^{-1} \tag{6}$$

To calculate $l^i_{\rm crit}$ use is made of the estimate in ref. 7.

The reaction probabilities $Y(i)$ for all incomplete-fusion reactions i are calculated according to eqns. (1)–(4), then this calculation is parametrized to the exponential factor (5), by variation of the parameter T_0. The angle-integrated cross-section for a particular incomplete-fusion channel then is

$$\sigma(i)=\pi\lambda\!\!\!^{-2}\sum_{l=0}^{l_{\max}}(2l+1)T_l\frac{P^i_l\exp[(Q^i_{gg}-\delta+\Delta E_c)/T_0]}{\sum_i P^i_l\exp[(Q^i_{gg}-\delta+\Delta E_c)/T_0]} \tag{7}$$

where $l_{\max}$ limiting the sum is defined as the largest angular momentum for which the colliding system enters the region where the total nucleus–nucleus potential is attractive.

2.3. Decay of heavy fragments

The energy spectra of reaction products reveal the presence of deep inelastic and quasi-elastic components. So, we present the excitation energies of primary heavy fragments as a superposition of two gaussian distributions. One of these corresponds to energy distribution in the quasi-elastic process, and the other to the deep inelastic process. The widths and relative contributions of these components are used as parameters. By means of random numbers the value of the excitation energy is determined. For different decay channels of the excited nucleus the maximum of the residual energy is defined in the following way:

$$E^{\max}_\nu=E^*-E_r-E_\nu-V_\nu;\quad E^{\max}_f=E^*-E_r-B_f$$

Here E^* is the excitation energy of the excited nucleus, E_r is its rotational energy, V_ν is the exit Coulomb barrier for a particle of the kind ν, E_ν is the kinetic energy of the particle and B_f is the fission barrier. For all $E^{\max}_\nu>0$ the kind of the emitted particle or γ ray is found. For partial widths of particle ν emission, the fission and the γ quanta emission, the following expressions have been used [8]:

$$\Gamma_\nu(E^*_{\rm H},I_{\rm H})\approx\frac{2(2s_\nu+1)}{\pi^2\hbar^3\rho_{\rm H}(U)}\int_{V_\nu}^{U-B_\nu}\sigma_{\rm inv}(E_\nu)\times\rho_K(U-B_\nu-E_\nu)E_\nu\,{\rm d}E_\nu \tag{8}$$

$$\Gamma_f(E^*_{\rm H},I_{\rm H})\approx[2\pi\rho_{\rm H}(U)]^{-1}\int_0^{U_s-B_\nu}\rho_s(U_s-B_f-\epsilon)\,{\rm d}\epsilon \tag{9}$$

$$\Gamma_\gamma(E^*_{\rm H},I_{\rm H})\approx\frac{3}{(\pi\hbar c)^2\rho_{\rm H}(U)}\int_0^U\sigma_{\gamma A}(E_\gamma)\rho_K(U-E_\gamma)E^2_\gamma\,{\rm d}E_\gamma \tag{10}$$

where U is the thermal energy of the nucleus. The inverse cross-section $\sigma_{\rm inv}$ is calculated within the optical model [9]:

$$\sigma_{\rm inv}=\begin{cases}\sigma_g c_1(1+c_2/E_\nu), & \nu=n\\ \sigma_g c_3(1+c_4V_\nu/E_\nu), & \nu={\rm p,\ d,\ t,\ ^3He},\ \alpha\end{cases} \tag{11}$$

Here σ_g is the geometrical cross-section, and r, c_1, c_2, c_3, c_4 are the parameters taken from ref. 9. In expression (9) the thermal energy U_s and rotational energy E^s_r are connected at the saddle point by the relation $U_s=E^*-E^s_r$. This form of the width Γ_f takes into account the change in the fission barrier of a rotating nucleus as far as $B_f(I)=B_f(0)-(E_r-E^s_r)$ (see details in ref. 10). In expression (10) for the partial width of the electric dipole radiation $\sigma_{\gamma A}$ is the photoabsorption cross-section.

To describe the level density as a function of the excitation energy, the well-known expression [11]

$$\rho(E^*)=\frac{\pi^{1/2}}{12}\frac{1}{a^{1/4}(E^*)^{5/4}}\exp[S(E^*)] \tag{12}$$

has been used. In eqn. (12) the dependence of the nucleus entropy S on the excitation energy E^* is determined by the relation

$$S=2at \tag{13}$$

by using the coupling of the nucleus temperature with its excitation energy:

$$E^*=at^2 \tag{14}$$

The parameter $a=\pi g_0^2/6$ of the level density is expressed through the density of single-particle states near the Fermi energy $g_0=g(E_f)=$ constant. The decrease in the influence of shell effects on the level density with increasing excitation energy is taken into account by the phenomenological expression [11]

$$a(E^*)=\tilde{a}[1+f(E^*)\ \delta W/E^*] \tag{15}$$

Here $f(x)=1-\exp(-\gamma x)$, and δW is the shell correction in the nucleus mass formula, $\tilde{a}=A(\alpha+\beta A)$ is the Fermi gas value of the level density parameter. The empirical values of the parameters $\alpha=0.134$ MeV^{-1}, $\beta=-1.21\times10^{-4}$ MeV^{-1}, $\gamma=6.1\times10^{-2}$ MeV^{-1} have been obtained from the analysis of the data on the level density with account taken of the contribution of the collective states to the total levels density:

$$\rho_{\text{tot}}(E^*)=K_{\text{rot}}K_{\text{vib}}\rho(E^*) \tag{16}$$

(see details in ref. 12).

After the determination of the de-excitation mode (if fission does not occur) the characteristics of the emitted particles or γ ray, namely their kinetic energy, orbital momentum and emission angle, were found. For the given kind of particle the simultaneous selection of E_ν, l and $\cos(\Theta)$ has been performed by using random numbers. Then by using the next random number they are rejected according to the three-dimensional probability density:

$$W[E_\nu, l, \cos(\Theta)] \approx l\, \exp\{2[a(E_k^*-E_\nu-(I^2+l^2)/2J + Il\cos(\Theta)/J)]^{1/2}\} \tag{17}$$

Here J is the moment of inertia of the excited nucleus. The azimuthal angle of the evaporated particle is taken in the coordinate system with the z axis parallel to I. The fission process is taken into account by the weight function

$$\text{FU}=\prod_{\nu=1}^{x}(1-\Gamma_{\text{f}}/\Gamma_{\text{tot}}) \tag{18}$$

This is convenient, in particular, for strongly fissionable nuclei. All the quantities are transformed to the centre of mass system of interacting nuclei and the characteristics of the residual nucleus, *i.e.*

$$I_{\text{f}}=I-l,\ E_{\text{res}}^*=E^*-B_\nu-E_\nu-(I_{\text{f}}^2-l^2)/2J,$$
$$A_{\text{res}}=A_{\text{CN}}-A_\nu;\ Z_{\text{res}}=Z_{\text{CN}}-Z_\nu \tag{19}$$

are calculated. Then the maximum residual energies of all emission processes and fission channels are calculated for this nucleus. Among the allowed values of $E_\nu^{\max}$ and $E_{\text{f}}^{\max}>0$ the next determination of the de-excitation type is performed. This is done while the condition $E_{\text{res}}>0$ is satisfied. Finally the heavy fragment production cross-section is calculated by summation over all the evaporation channels giving a substantial contribution to the yield of isotope with given A and Z. The gathering of the required statistics for the calculation of different reaction characteristics has provided about 5% calculation accuracy.

The disappearance of shell effects in the nuclear level density ρ with increasing E^* is accounted for in the framework of the Fermi gas model using a phenomenological dependence [11, 12].

3. Conclusions

The parameters of the model used in the calculation are the parameter Δ_l of the angular momentum distribution, the slope T_0 of the exponent in eqn. (5) and nuclear parameters r_0 of exit and entrance reaction channels. For realistic values of these parameters a rather good agreement with the experimental data on the production of some isotopes of heavy actinides has been obtained (Fig. 2).

The presented calculations of the production cross-sections for heavy fragments in incomplete-fusion reactions are preliminary. Reasonable values of model parameters permit satisfactory estimates of heavy actinide yields to be obtained. For more realistic cross-section estimates a more accurate experimental data analysis is needed.

A microscopic approach to the calculation of probabilities for the production of heavy transfer products formulated in ref. 4 makes it possible to avoid using a phenomenological dependence for $Y(i)$ which is well known only for a narrow energy region above the fusion barrier.

For calculations in which account is taken of the decay of heavy products accurate knowledge of such characteristics as fission barrier and its dependence on temperature is needed. For the heavy excited fissioning systems the liquid-drop fission barrier can be comparable with the nuclear temperature. In this respect a question arises concerning the application of statistical calculations in this region of nuclei. Thus, taking into account such effects as nuclear viscosity becomes desirable.

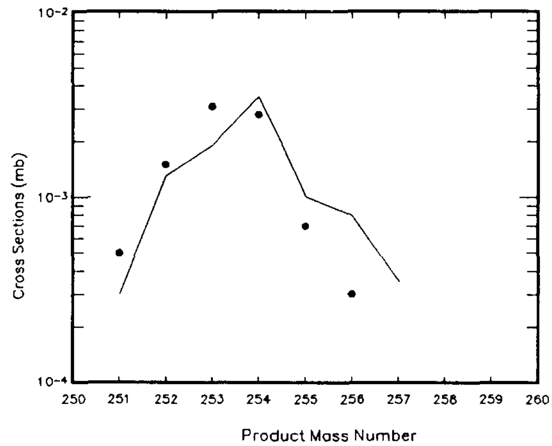

Fig. 2. Calculated (——) and experimental (●) results for the cross-section of the reaction $^{18}O+^{248}Cm\rightarrow Fm$.

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